A metaworld: metastability in metacommunities

A. Franc & N. Peyrard

INRA BioGeCo & MIAT

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Some points which will be addressed

Community Structure of Tropical Rainforests

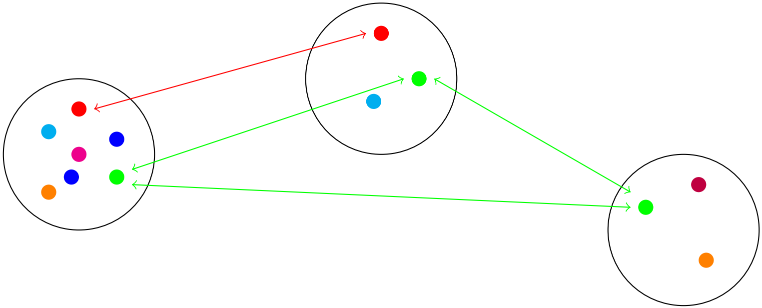
Bats and rabbies in French Guiana

- bats live in caves
- males and females are distinguished (different dispersal)
- they can be healthy or infected
- a cave is a patch: island model
- dispersal between caves distance dependent, by males

What is a metacommunity?

Definition (Leibold *& al.*, Ecology Letters, 2004)

A metacommunity is a set of communities located at some sites, or patches, connected by dispersal.



Modeling

- species interact within one patch
- species disperse along a graph connecting patches

Which questions to address?

Context: longstanding question in ecology (Clements (1936), Gleason (1926), Hubbel (2001))

What is the role of local adaptation and dispersal in shaping communities?

- here: which is the role of the shape of dispersal networks?

$\Rightarrow$ Geometry of networks

The within patches processes having been selected, what is the impact of the structures of dispersal graphs on the outcome of the model

- the quasi-stationary distribution
- the bifurcations (or phase transitions)

• Simplification

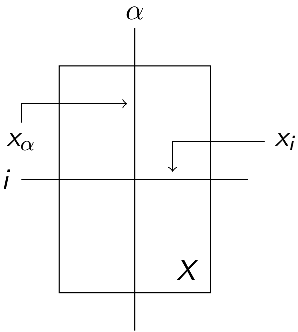
Is it possible to exhibit relevant results with simplifications like mean field approximation?

Modeling metacommunities as reaction-diffusion processes on graphs

Formalisation

space	discrete (patches)
time	continuous
state	binary matrix
reaction	intra-patch dynamics one reaction process per patch
diffusion	between patches dispersal one dispersal graph per species

State of the metacommunity



The model: a times continuous Marckov Chain

Master equation ($2^{np} \times 2^{np}$)

$$\mathbb{P}(X^{t+dt} = x) = \sum_{x'} \mathbb{P}(\underbrace{X^{t+dt} = x \,|\, X^t = x'}_{\text{one event}}) \times \mathbb{P}(X^t = x')$$

an event: reaction (within patch interaction) or diffusion (between patches disperal)

Reaction phase ($2^p \times 2^p$)

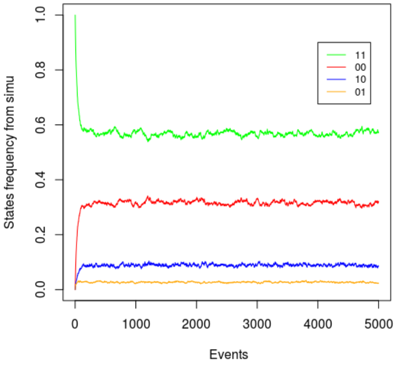
$$\mathbb{P}(X_i^{t+dt} = x \,|\, X_i^t = x') = \begin{cases} R_{i,xx'}\,dt & \text{if} \quad x \subsetneq x' \\ 1 - \sum_{x \subsetneq x'} R_{i,xx'}\,dt & \text{if} \quad x = x' \end{cases}$$

Dispersal phase ($2^n \times 2^n$)

For each species α, dispersal along an edge of a weighted graph

$$\mathbb{P}(A_\alpha^{t+dt} = a \,|\, A_\alpha^t = a') = \begin{cases} D_{\alpha,aa'}\,dt & \text{if} \quad a' \subsetneq a \\ 1 - \sum_{a' \subsetneq a} D_{\alpha,aa'}\,dt & \text{if} \quad a = a' \end{cases}$$

Simulations ...



Erdös-Renyi random graph ; 50 nodes; edge density = 0.2 ; reaction as predator-prey ; dispersal 0.2 & 0.8

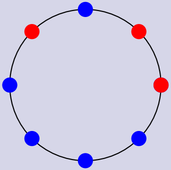
Metastability or quasi-stationary distribution

heuristically ...

For a Markov chain with an absorbing state:

- the asymptotic state (or equilibrium) is the absorbing state
- however the chain can wander during a very long (not infinite) time over an observed subset of non absorbing states

Contact Process



Metastable state

$$\rho_t = \frac{1}{n} \sum_i x_i^t$$

Some notions (here in discrete time)

Killing time

$$T_0 = \inf \{t \in \mathbb{R} \;:\; x^t = x_\infty\}$$

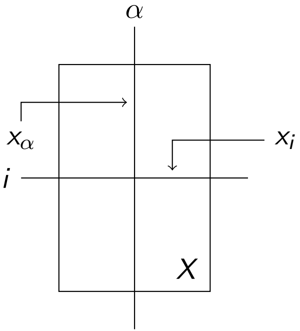
Conditionally Invariant Distribution

$$P = \begin{pmatrix} 1 & a \\ \mathbf{0} & P^* \end{pmatrix} \qquad \begin{pmatrix} 0 \\ u^* \end{pmatrix} \xrightarrow{P^t} \begin{pmatrix} 1-\rho^t \\ \rho^t u^* \end{pmatrix} \qquad \text{if} \quad P^* u = \rho u^*$$

Quasi-stationary distribution

$$u^* \quad s.t. \quad P^* u^* = \rho\, u^*, \qquad \mathbb{E}(T_0) = \frac{1}{1-\rho}$$

Computational Complexity



Computational Complexity

- Dynamics of a metacommunity is modeled as a Markov Chain on a state space Ω with $|\Omega| = 2^{np}$
- the size of matrix P is $2^{np} \times 2^{np}$

Pollet's contribution (discrete time too)

Block diagonal form of transition matrix

$$P = \begin{pmatrix} 1 & A_{11} & \dots & A_1 q \\ 0 & P_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & A_{q-1,q} \\ 0 & \dots & \dots & P_q \end{pmatrix}$$

$\Longrightarrow$ to compute quasi-stationary distribution, it suffices to compute largest eigenvalues of matrices P_k (circulation classes).

An observation

There is a one to one correspondence between circulation classes and subsets of species.

Still some limits from computation complexity

Size of circulation classes

If $p = 2$, there are 4 blocks, of size respectively

S	size
00	1
01	$2^n - 1$
10	$2^n - 1$
11	$(2^n - 1)^2$

bock sizes for $n = 10$

1×1	1×511	1×511	$1 \times 1\,046\,529$
0	511×511	511×511	$511 \times 1\,046\,529$
0	**0**	511×511	$511 \times 1\,046\,529$
0	**0**	**0**	$1\,046\,529 \times 1\,046\,529$

Mean-Field approximation: mean degree

(back to continuous time)

with words ...

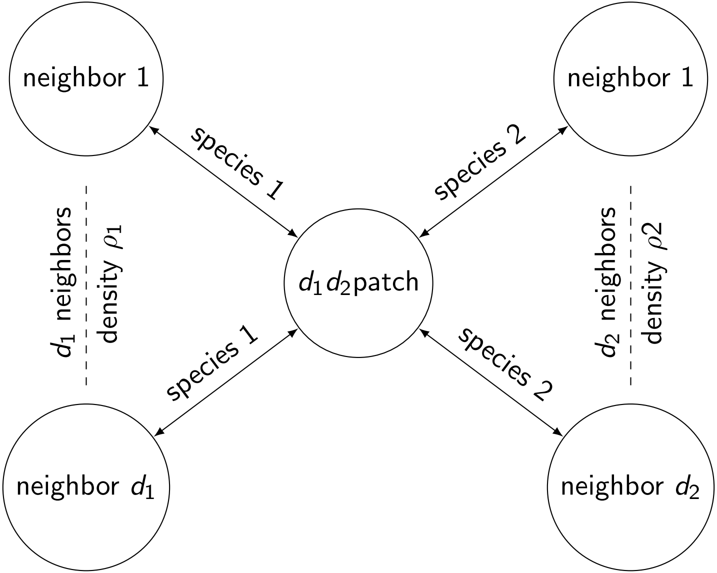
Each species α in site i "sees" $z_{\alpha i}$ neighbor patches occupied or not by species α with global probability

$$\rho_\alpha^t = \frac{1}{n} \sum_i x_{i\alpha}^t$$

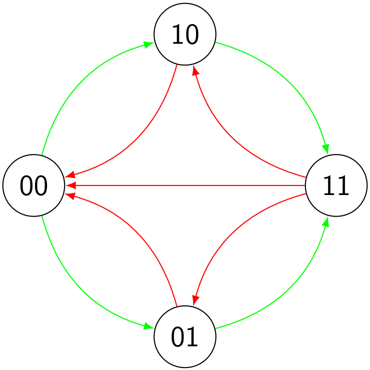
with equations ...

$$\begin{cases} \mathbb{P}(X_{i\alpha}^{t+dt} = 1 \,|\, X_{i\alpha}^t = 1) = 1 \\ \mathbb{P}(X_{i\alpha}^{t+dt} = 1 \,|\, X_{i\alpha}^t = 0) = c\, d_{\alpha i}\, \rho_\alpha\, dt \end{cases}$$

Mean-Field approximation: with degree distribution (1/2)



Mean-Field approximation: with degree distribution (2/2)



state at time $t + dt$	event $(01 \rightarrow \dots)$	rate
11	reaction	$R_{01,11}\,dt$
00	dispersal of species 2	$d_2 c_2 \rho_2^t\,dt$
01	no reaction	$1 - R_{00,01}\,d_1 t$
01	no dispersal of species 1	$1 - d_1 c_1 \rho_1^t\,dt$

The role of space: graph families

Geometry of networks

The within patches processes having been selected, what is the impact of the structures of dispersal graphs on the outcome of the model

- the quasi-stationary distribution
- the bifurcations (or phase transitions)

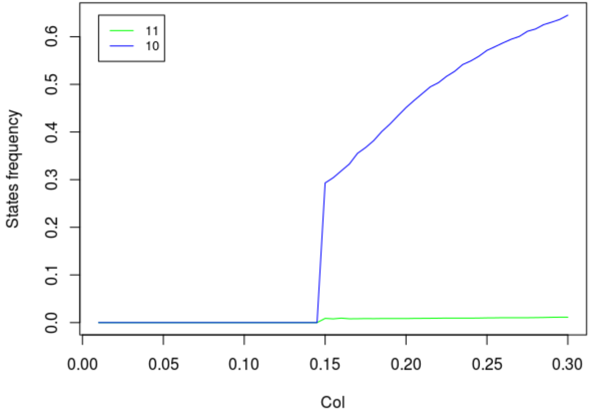
Graph families

- a graph with n nodes can have $2^{\frac{n(n-1)}{2}}$ edge patterns
- $\Longrightarrow$ a simplification with the notion of *graph family*
- which is a rule to build a graph (with random)

Exemples

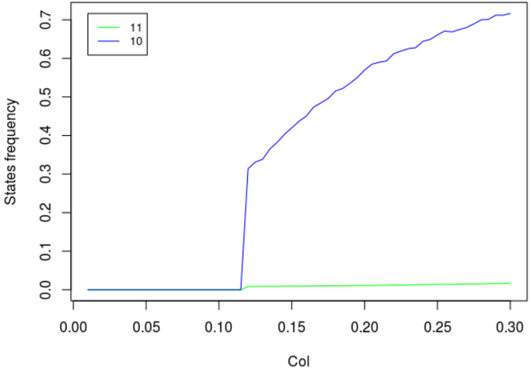
- Erdös-Rényi random graph with degree probability p
- Geometric random graph
- Star graph

Grid or a lattice



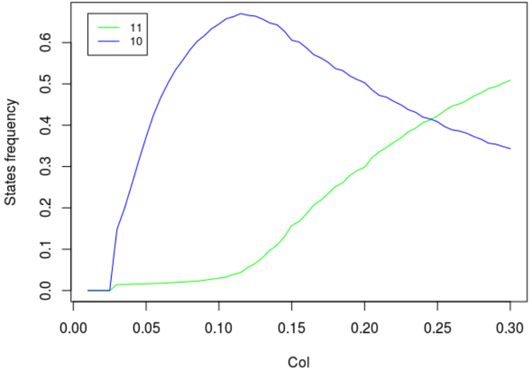
A host/parasite system (one colonization rate) ; the dispersal graph is a square grid of size 7×7; healthy host in blue ; infected host in green.

Geometric Random Graph



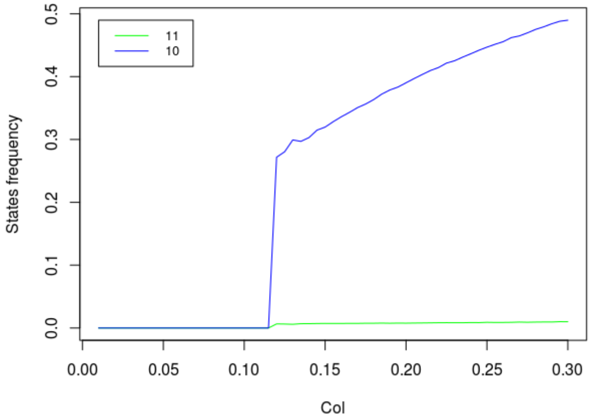
A host/parasite system (one colonization rate) ; the dispersal graph is a Geometric Random Graph ; 50 nodes ; healthy host in blue ; infected host in green.

Geometric Random Graph



A host/parasite system (one colonization rate) ; the dispersal graph is a Geometric Random Graph ; 100 nodes ; healthy host in blue ; infected host in green.

A star



A host/parasite system (one colonization rate) ; the dispersal graph is a star ; 50 nodes ; healthy host in blue ; infected host in green.

Conclusions & perspectives

Methods

- Some work remains to be done on mean-field approximation
- can be extended to pair approximation, Bethe, Kikuchi, . . .

Ecological models

The model is versatile: can be tuned with some work to be relevant in different situations:

- trophic web
- epidemiology (compartiment models as mean field)
- biogeography (dispersal and competition between trees)

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